Therefore, we have the following equivalences:

$$\begin{aligned}
&\frac{a}{b} = 1 + \sqrt{2\left(\frac{c}{b}\right)^2 - 1} \\
\iff \quad & \sqrt{5} - 1 = 1 + \sqrt{\frac{7 - 3\sqrt{5}}{3 + \sqrt{5}}} \\
\iff \quad & (\sqrt{5} - 2)^2 = \frac{7 - 3\sqrt{5}}{3 + \sqrt{5}} \\
\iff \quad & (9 - 4\sqrt{5})(3 + \sqrt{5}) = 7 - 3\sqrt{5} \\
\iff \quad & 7 - 3\sqrt{5} = 7 - 3\sqrt{5},
\end{aligned}$$

which is true and our proof is complete.

4009. *Proposed by George Apostolopoulos.*

Let m_a, m_b, m_c be the lengths of the medians of a triangle ABC. Prove that

$$\frac{1}{m_a} + \frac{1}{m_b} + \frac{1}{m_c} \le \frac{R}{2r^2},$$

where r and R are inradius and circumradius of ABC, respectively.

We received eleven solutions, of which ten were correct. We present two solutions.

Solution 1, by Arkady Alt.

Let F, s and h_a, h_b, h_c be the area, semiperimeter, and altitudes of the triangle. Since $m_x \ge h_x, x \in \{a, b, c\}$ and

$$\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{a}{2F} + \frac{b}{2F} + \frac{c}{2F} = \frac{s}{2F} = \frac{1}{r}$$

then

$$\frac{1}{m_a} + \frac{1}{m_b} + \frac{1}{m_c} \le \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r} \le \frac{R}{2r^2}$$

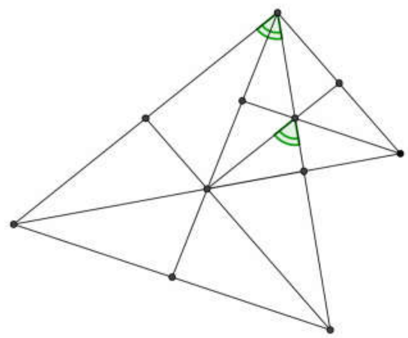
because

$$\frac{1}{r} \le \frac{R}{2r^2} \iff 2r \le R,$$

by Euler's Inequality.

Solution 2, by Edmund Swylan.

We take it as known that the triangle with side lengths $2m_a$, $2m_b$, $2m_c$ has medians of lengths $\frac{3}{2}a$, $\frac{3}{2}b$, $\frac{3}{2}c$. (See the drawing below.)



Let the area of $\triangle ABC$ be F. The area of the big triangle is then $3F$. Let the altitudes of the big triangle be H_a, H_b, H_c.

We have that $\dfrac{6F}{2m_x} = H_x$ and $H_x \leq \dfrac{3}{2}x$, for each $x \in \{a, b, c\}$. Therefore,

$$3F(\frac{1}{m_a} + \frac{1}{m_b} + \frac{1}{m_c}) \leq \frac{3}{2}(a+b+c);$$

equality occurs if and only if the big triangle, and consequently $\triangle ABC$ too, is equilateral. Finally,

$$\frac{3}{2}(a+b+c) = 3F\frac{1}{r} \leq 3F\frac{1}{r}\frac{R}{2r} = 3F\frac{R}{2r^2};$$

equality occurs if and only if $\triangle ABC$ is equilateral.

4010. *Proposed by Ovidiu Furdui.*

Let $f : [0, \frac{\pi}{2}] \to \mathbb{R}$ be a continuous function. Calculate

$$\lim_{n\to\infty} n \int_0^{\frac{\pi}{2}} \left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)^{2n} f(x)\mathrm{d}x.$$

There were eight submitted solutions for this problem, all of which were correct. We present two solutions.

Solution 1, by the group of M. Bello, M. Benito, Ó. Ciaurri, E. Fernández, and L. Roncal, expanded slightly by the editor.

The value of the required limit is $\dfrac{1}{4}\left(f(0) + f\left(\dfrac{\pi}{2}\right)\right)$. Indeed, if we denote by L the limit, then from the identity

$$\frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} = \tan\left(\frac{\pi}{4} - x\right),$$